IMAT ASM'S ANNUAL MEETING

INTERNATIONAL MATERIALS, APPLICATIONS & TECHNOLOGIES 2025

SU(2) symmetry breaking:

A deeper look at solidification and resulting thermophysical properties

PSDK XV: Phase Stability and Diffusion Kinetics III

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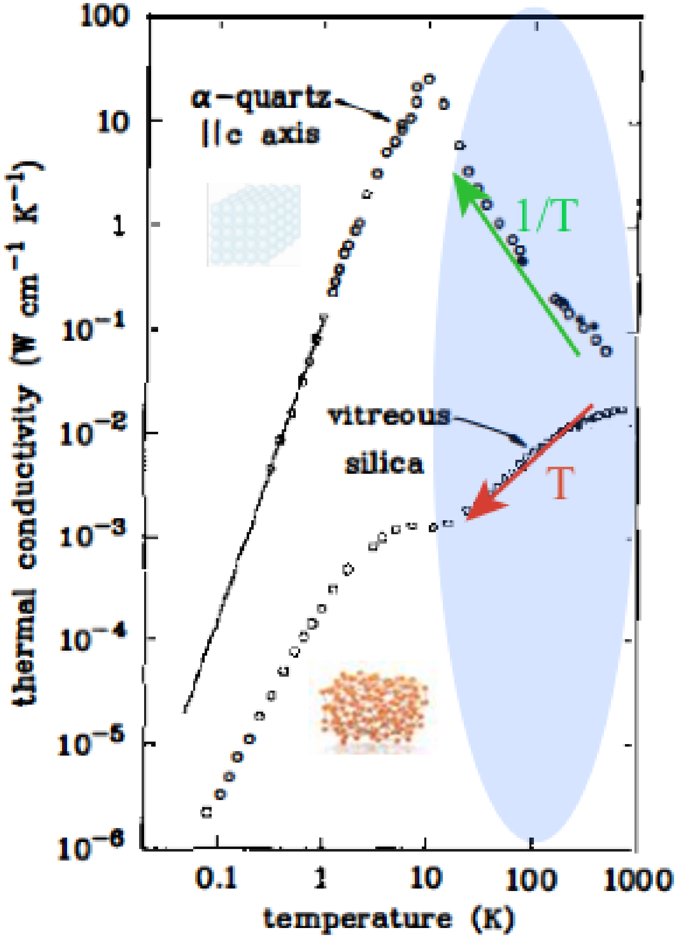
Thesis Advisor: **Prof. David E. LAUGHLIN**

October 20, 2025

Things I Learned While Being Advised by Prof. David E. Laughlin

Approaching a unified description of solid state thermal conductivity!



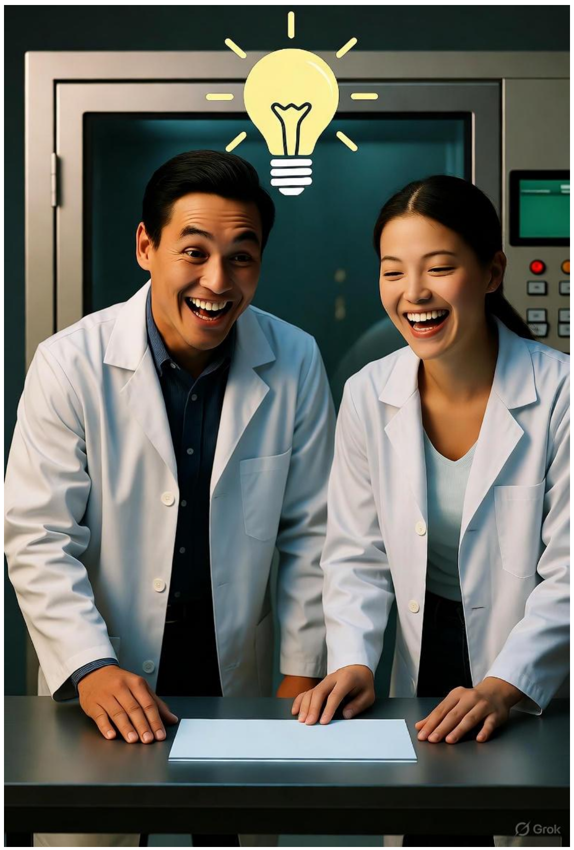
Eucken (1911) first pointed out that **κ** of crystals and glasses have inverse temperature dependences ($T > 50$ K)

"The deepest and most interesting unsolved problem in solid state theory is probably the theory of the nature of glass and the glass transition. This could be the next breakthrough in the coming decade." P W Anderson, Science (1995)

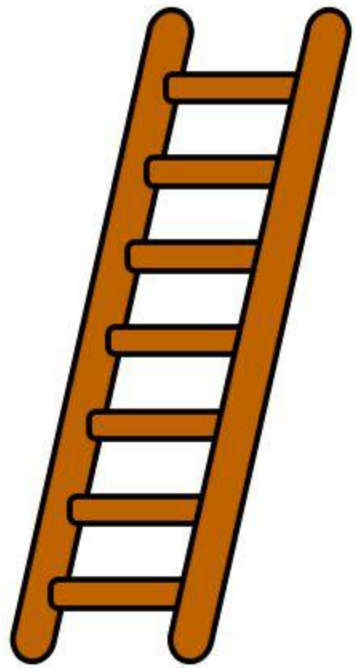
"for now we see through a glass darkly"

[1] Eucken, 1911, **10.1002/andp.19113390202**.
[2] Cahill, Pohl, 1988, 10.1146/annurev.pc.39.100188.000521.
[3] PW Anderson, Through the Glass Lightly, Science (1995).

Standing on the shoulders of giants

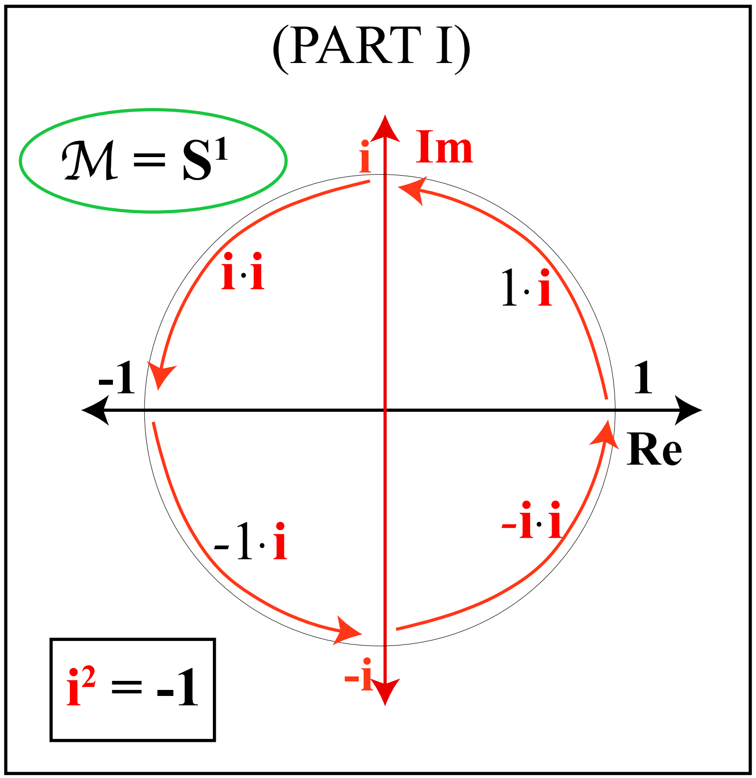


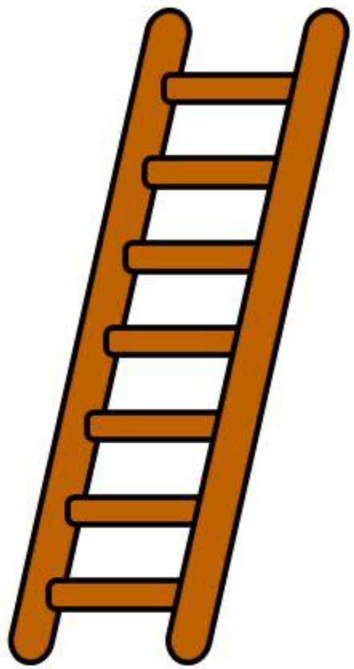
Part I

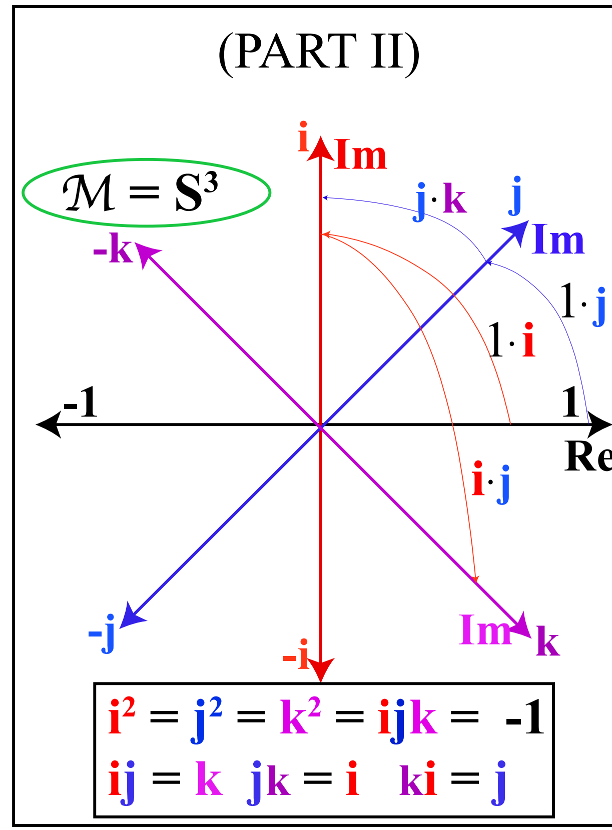


Part II

Pursuit of Analogies… PART I

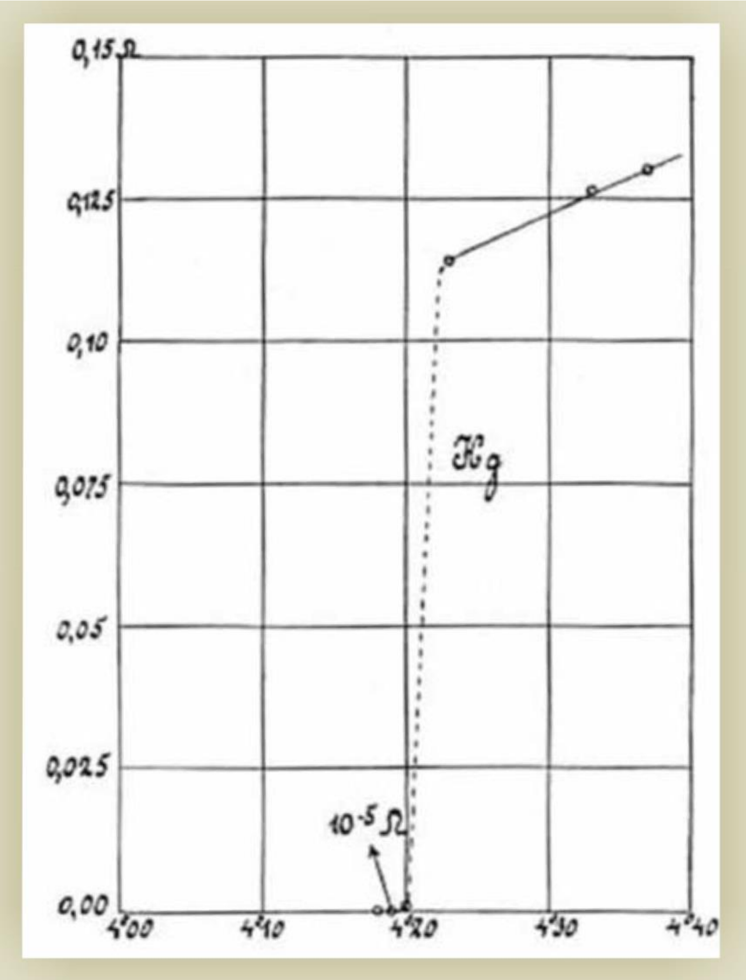






Superconductivity & Superfluidity

Transport Properties: Ω and η → 0



In 1911 (Onnes), it was observed that (below a critical temperature) certain metals show zero electrical resistance

Similarly, in 1937 (Kapitza) it was observed that the viscosity of liquid helium-4 falls to 0 below the λ-point.

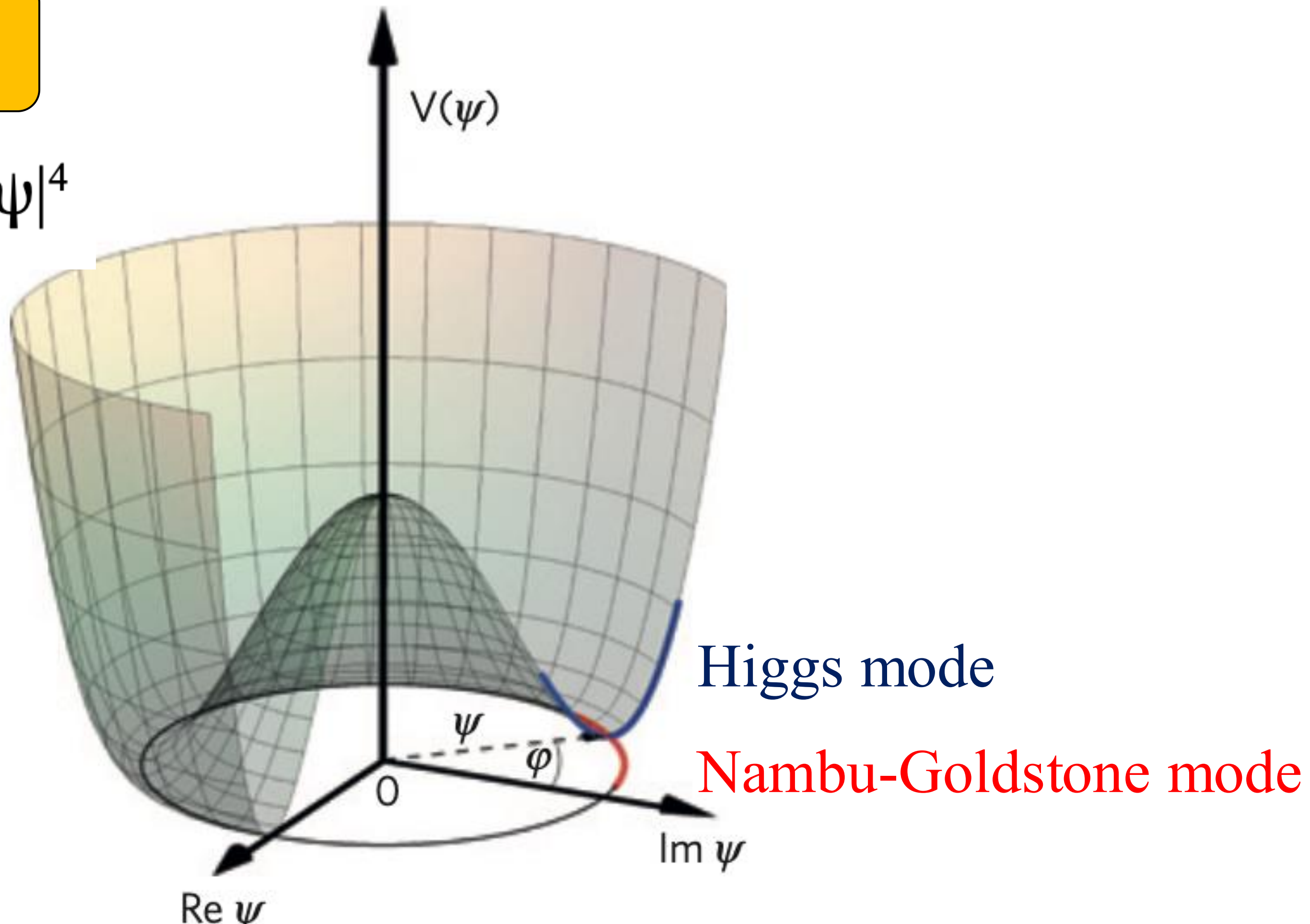
[4] Onnes, Phys. Lab. 12, 120 (1911).
[5] Kapitza, Nature, Vol 141 (1938).

Superconductivity & Superfluidity

Complex wavefunction order parameter

O(2) ~ U(1) ~ S^1

$$F_{superfluid} = a|\psi|^2 + b|\psi|^4$$



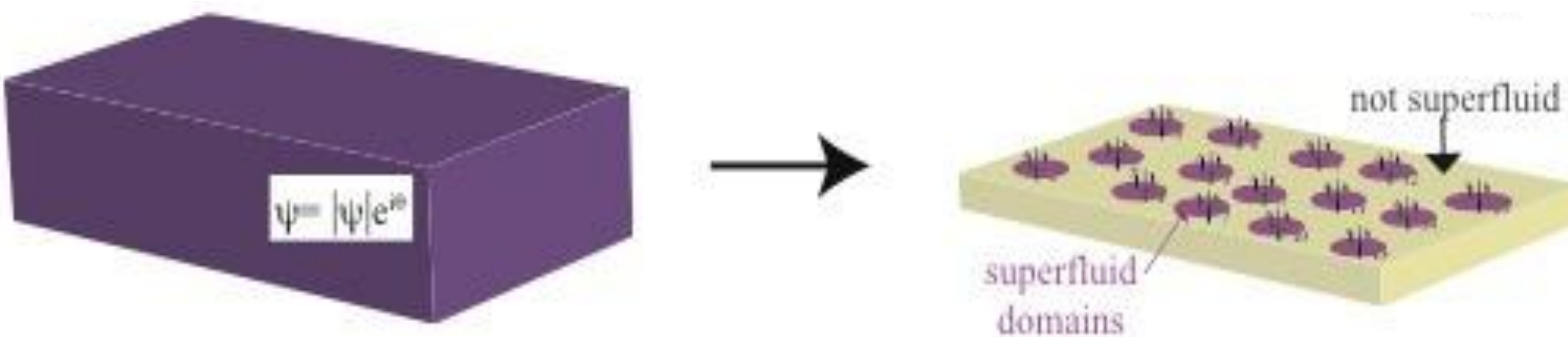
[6] Ginzburg, Landau, J. Exptl. The. Phys. 20, 1064 (1950)

Importance of Physical Dimensions: *Mermin-Wagner & Undercooling*

Mermin-Wagner theorem: rigorously proves the inability of *isotropic (i.e., continuous symmetry)* systems cannot undergo *spontaneous symmetry breaking* at finite temperatures (T_λ or T_{BCS}) in 2D/1D

In 3D:
Global ordering @ T_{BCS}

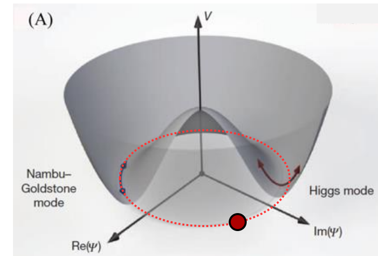
In 2D/1D:
Phase separated for $T < T_{BCS}$



[7] Mermin Wagner, "Absence of Ferromagnetism or Antiferromagnetism in 2D or 2D isotropic Heisenberg Models," PRL (1966).
[8] Gantmakher Dolgopolov, "Superconductor-insulator quantum phase transition," Physics Uspekhi (2010).

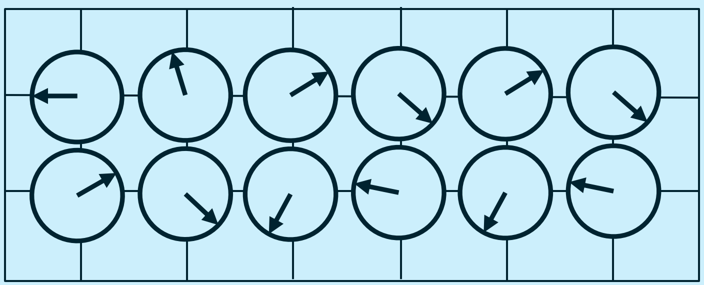
Importance of Physical Dimensions: $S^1 \in \mathbb{R}^n$

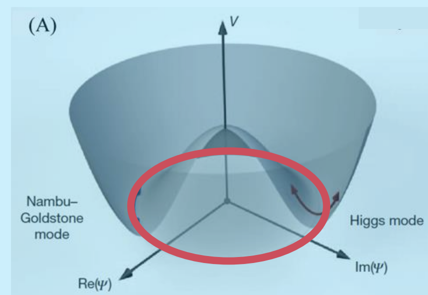
$S^1 \in \mathbb{R}^3$ bulk superfluidity/conductivity



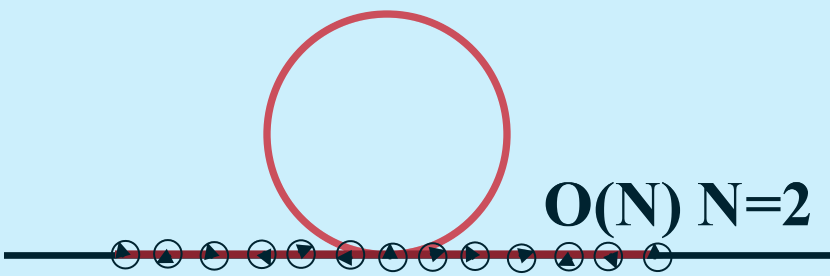
$S^1 \in \mathbb{R}^2$

O(N) N=2 in 2D





$S^1 \in \mathbb{R}^1$



O(N) N=2 in 1D

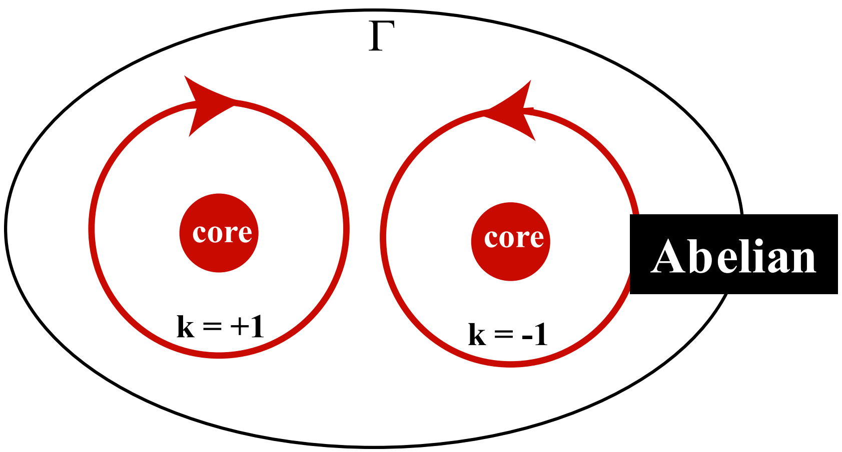
Restricted dimensions

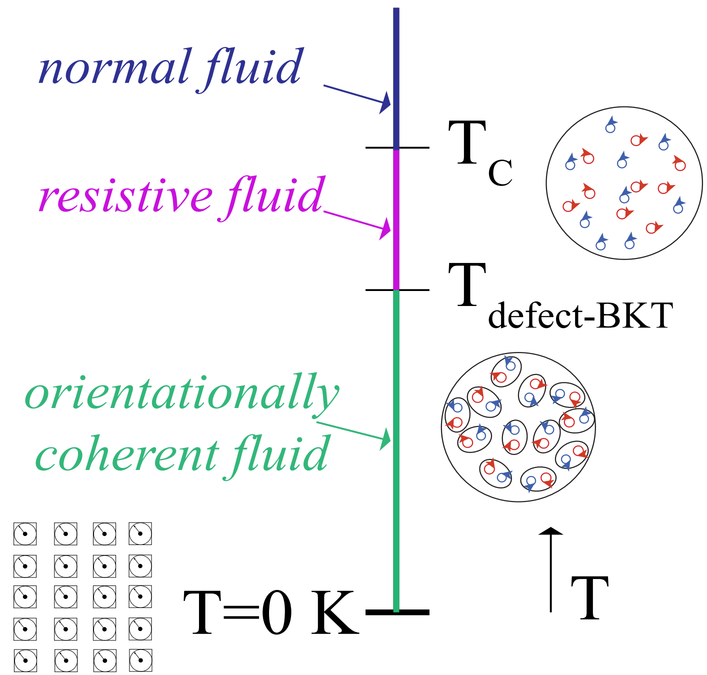
Restricted Dimensions:

Berezinskii-Kosterlitz-Thouless Transition

Berezinskii-Kosterlitz-Thouless

Understood in terms of pairing of $\pi_1(S_1) = Z$ topological defects!





[9] Berezinskii, V. L.. *Sov. Phys. JETP*, *32*(3), (1971).
[10] Kosterlitz, J. M., & Thouless, D. J., *J. Phys. C: Solid State Physics*, *6*(7), (1973).

Restricted Dimensions:

Potential for Dual Ground States - JJA

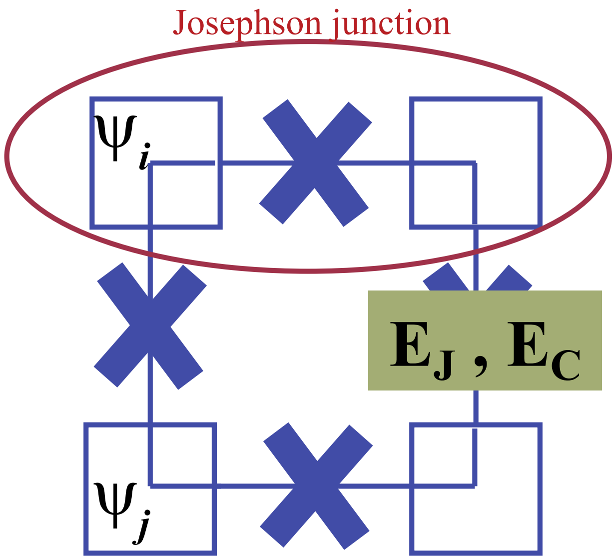
$\psi = \sqrt{n_S}e^{i\theta}$ has both amplitude and phase. This leads to an uncertainty relation: $\Delta n_S \Delta\theta \geq 1$. i.e., Δn_S *and* $\Delta\theta$ cannot be mutually minimized.

<u>Josephson Junction Array Hamiltonian:</u>

$$\widehat{\mathrm{H}} = 4E_C\left(\widehat{N} - n_g\right)^2 - E_J \cos\widehat{\emptyset}$$

E_C : charging energy ($E_C = e^2/2C$)

E_J : Josephson coupling energy ($E_J = I_C\Phi_0/2\pi$)



Operators satisfy commutation relation: $[\widehat{N}, \widehat{\emptyset}] = \hat{\imath}$

[11] Anderson, "Considerations on the flow of superfluid helium," Rev. Mod. Phys. (1966).
[12] Mooij, Nazarov, Quantum phase slip junctions (2005).
[13] Vinokur et. al., Superinsulator & quantum synchronization (2008).

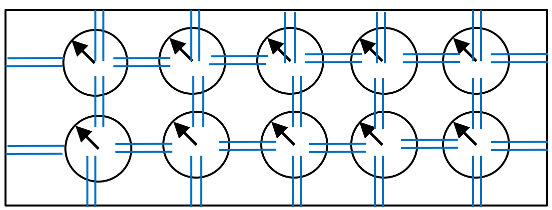
Restricted Dimensions:
Non-thermal tuning parameter (g)

$\hat{H}(g) = g\hat{T} + \hat{V}$

Non-thermal *tuning* parameter $\boldsymbol{g = E_C/E_J}$ mediates the ratio of $\hat{T}$ and $\hat{V}$ energies which cannot be minimized simultaneously.

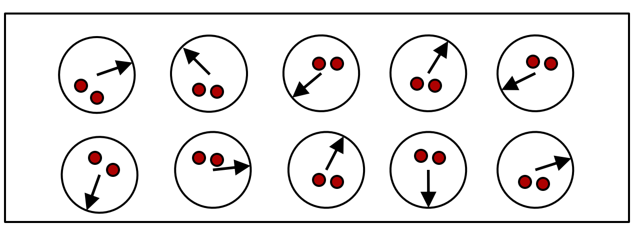
- **quantum phase transition at $\mathbf{g_c}$**

<u>Small $g < g_C$ limit: $\hat{V}$</u> *favors orientationally-ordered ground state*



- **minimization of $\Delta\boldsymbol{\theta}$**

<u>Large $g > g_C$ limit: $\hat{T}$</u> *favors orientationally-disordered ground state*



- **minimization of $\Delta \mathbf{n_S}$**

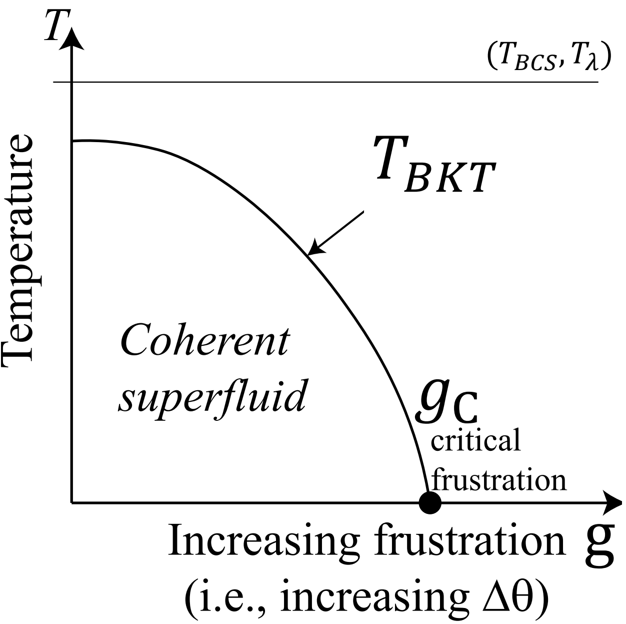
[14] Sachdev, "Quantum phase transitions," (2007).
[15] Fazio Schon, "Charge and vortex dynamics in arrays of tunnel junctions," PRB (1990)

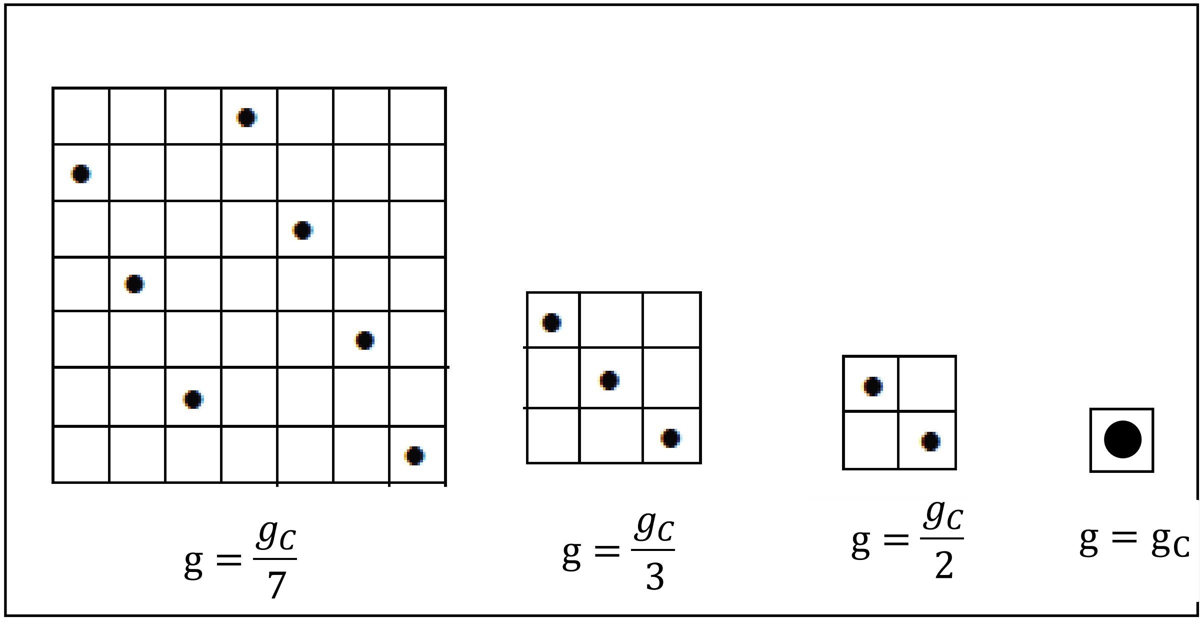
Restricted Dimensions:

Frustration in the vicinity of QPT

$0 < g < g_C$: the system is *frustrated*. A finite density of misorientational topological defects ($\Delta\theta$) are forced into the ground state – *in a manner satisfying the 3rd law.*

Coherent superfluidity is destroyed as the flux lattice length scale approaches the coherence length, decomposing a homogeneous state into individual superfluid islands.





[16] Teitel Jayaprakash, "Josephson-Junction Arrays in Transverse Magnetic Fields," PRL (1983).

Restricted Dimensions:

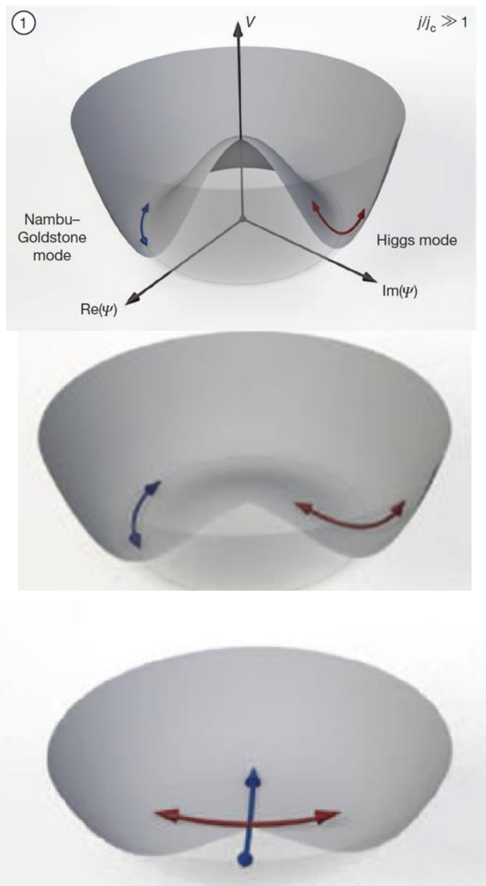
Free-Energy Function in vicinity of QPT

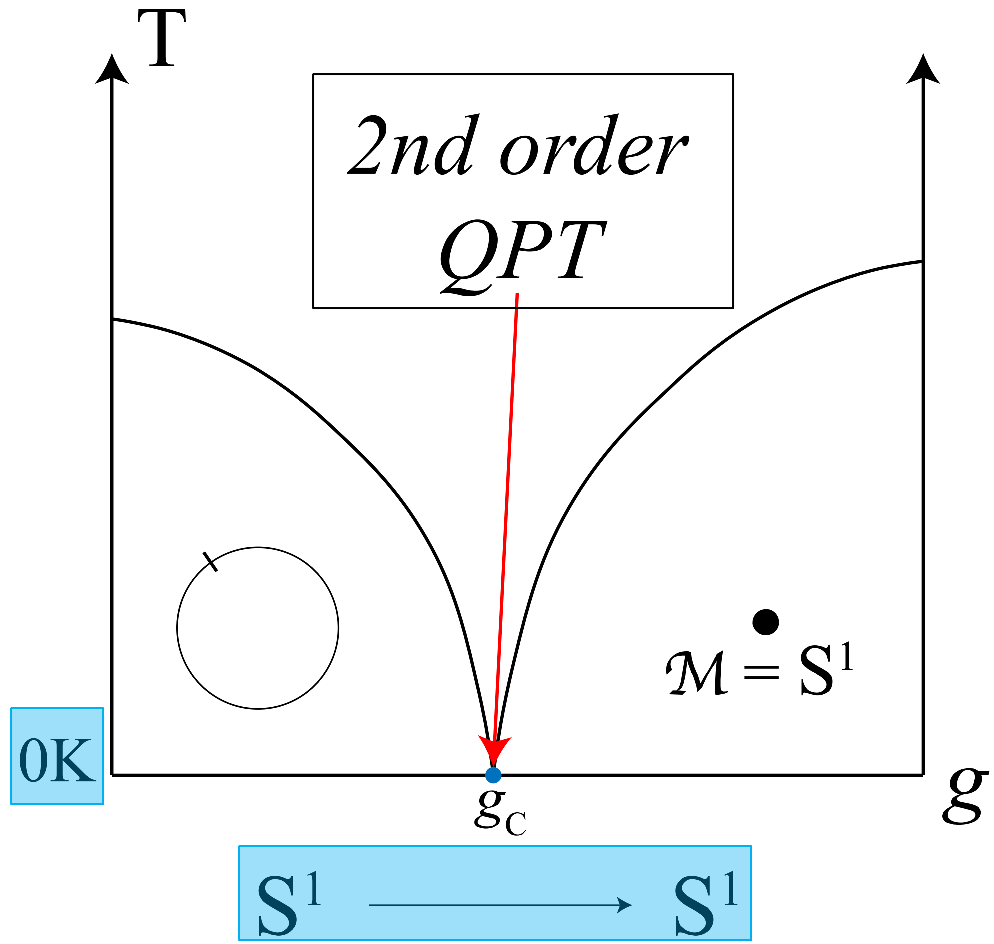
In 2D/1D complex ordered systems, $\mathcal{M}$ is <u>continuous</u> across the QPT

$g = 0$

$g < g_C$

$g = g_C$

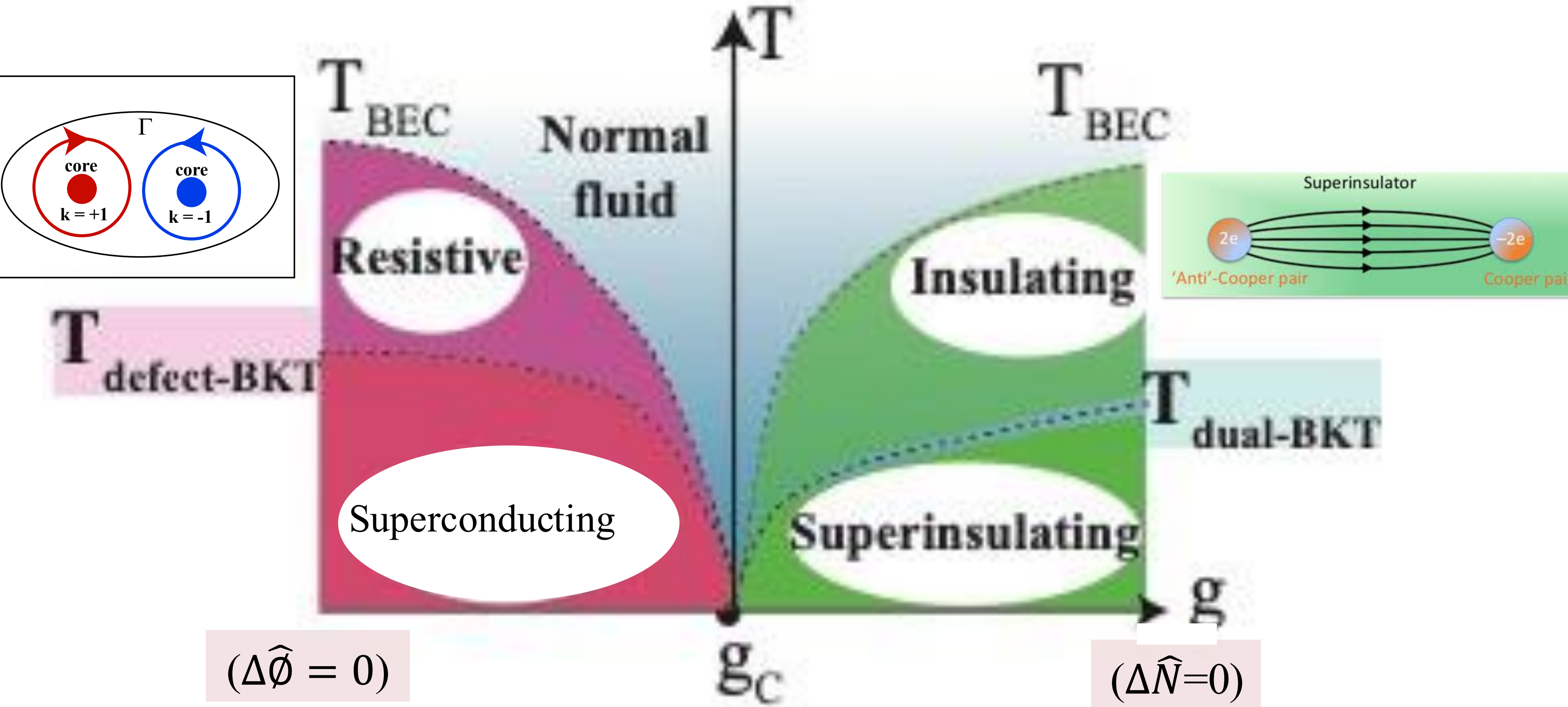




[17] Endres, et al. "The 'Higgs' amplitude mode at the 2D superfluid/Mott insulator transition." *Nature* (2012).
[18] Sherman et al., The Higgs mode in disordered superconductors close to a quantum phase transition, Nature phys Vol 11 (2015).

Restricted Dimensions:

Superconductor-Superinsulator Phase Diagram



[19] Baturina, Vinokur, Superinsulator–superconductor duality in two dimensions (2013).

Restricted Dimensions:
Dual transport properties

Electrical resistance as a function of temperature varies in dually across the superconductor-to-superinsulator transition.

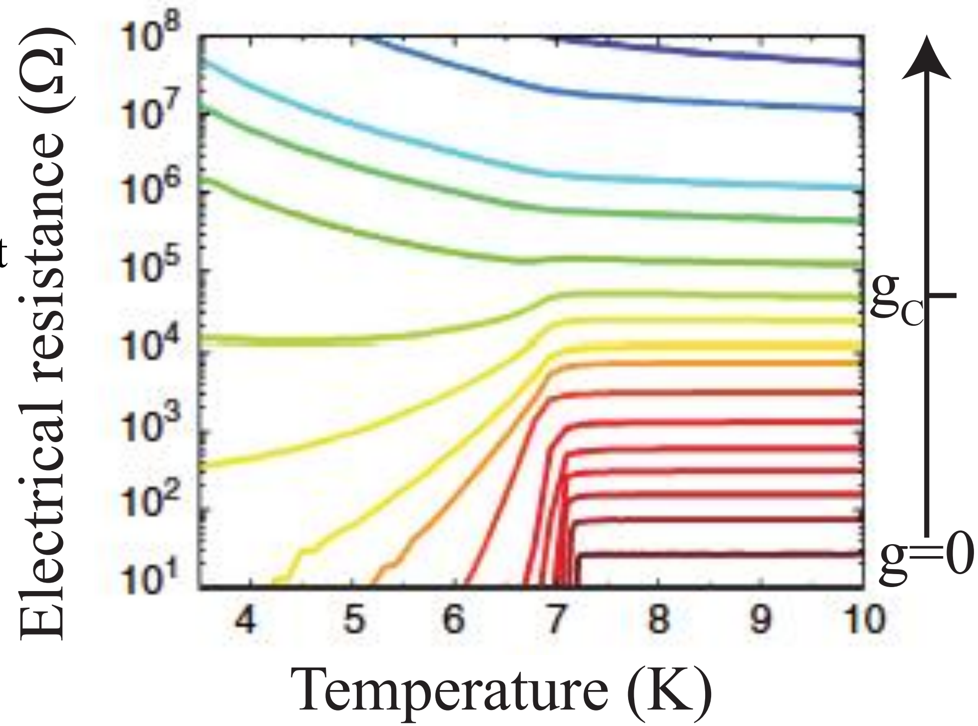
In the superinsulator phase:

- $\Delta n \rightarrow 0$ then $\Delta\theta \neq 0$
- By Josephson Equations, this implies that a finite voltage may develop

$$\frac{d\theta}{dt} = \left(\frac{2e}{\hbar}\right)V = \left(\frac{2\pi}{\Phi_0}\right)V$$

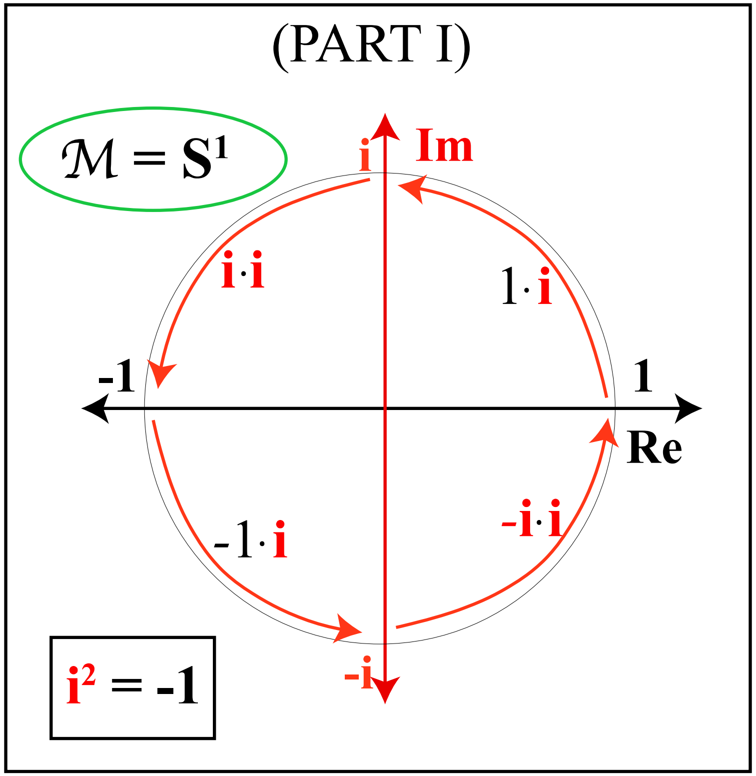
- However, current is blocked due to prevention of flow of Cooper pairs:

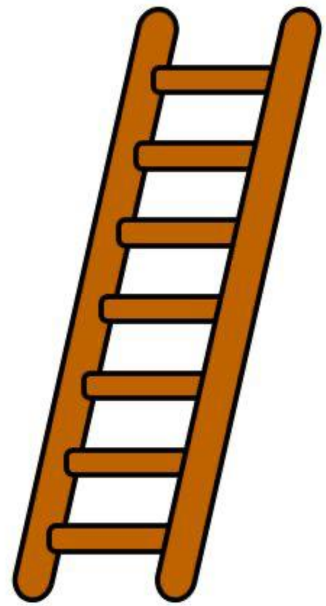
$$R \rightarrow \infty$$

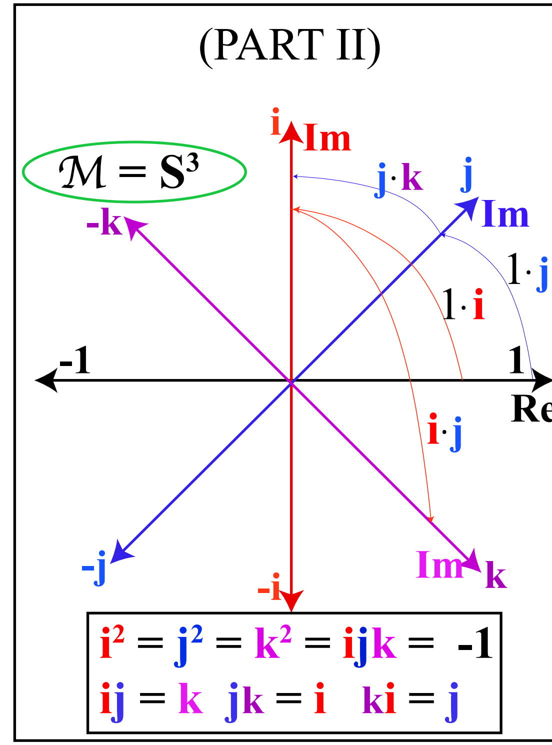


[20] Poran et al., "Quantum criticality at the superconductor-insulator transition revealed by specific heat measurements" Nat. Comm. (2017).

Pursuit of Analogies… PART II







H has 4-dim: $q = a + b\,\hat{\imath} + c\,\hat{\jmath} + d\,\hat{k}$

ℂ has 2-dim: $z = a + b\,\hat{\imath}$

Orientational Order Parameter:

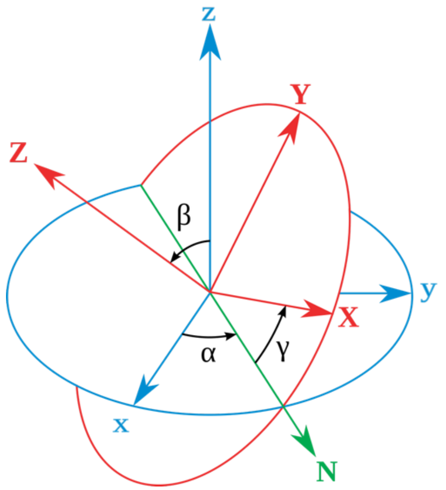
SO(3) Euler Angles & SU(2) Quaternions

G= SO(3) characterizes rotational d.o.f. available to atomic liquid.

G =SO(3) is not simply connected, i.e., $\pi_1(SO(3)) = Z_2$

Topological properties of condensed matter systems are understood by considering symmetry breaking of *simply connected (universal covering) groups*.

Unit quaternion group G=SU(2) $\cong S^3$ is *universal covering group* of G=SO(3).



The *lift* of H $\in SO(3)$ into $SU(2)$ is twice the size and is known as the binary group H$' \in SU(2)$

$$\psi(\mathrm{r}) \rightarrow \psi(\mathrm{r})\mathrm{e}^{\hat{\tau}\alpha}, \hat{\tau}^2 = -1$$

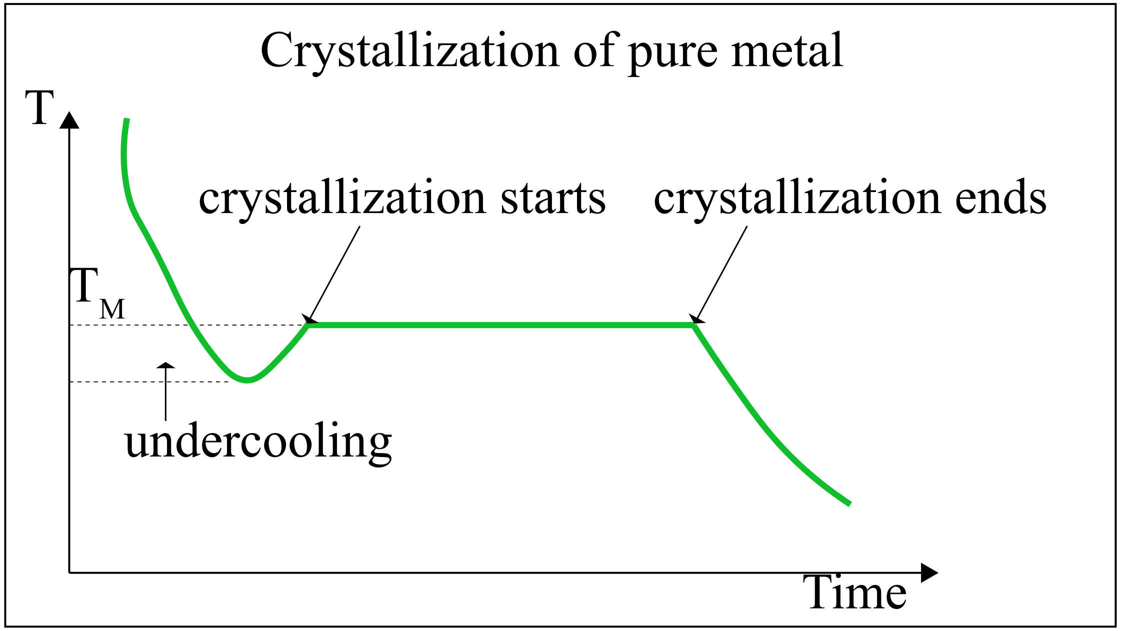
$$\hat{\tau} = \cos\beta\hat{\imath} + (sin\beta cos\gamma)\hat{\jmath} + (sin\beta sin\gamma)\hat{k}$$

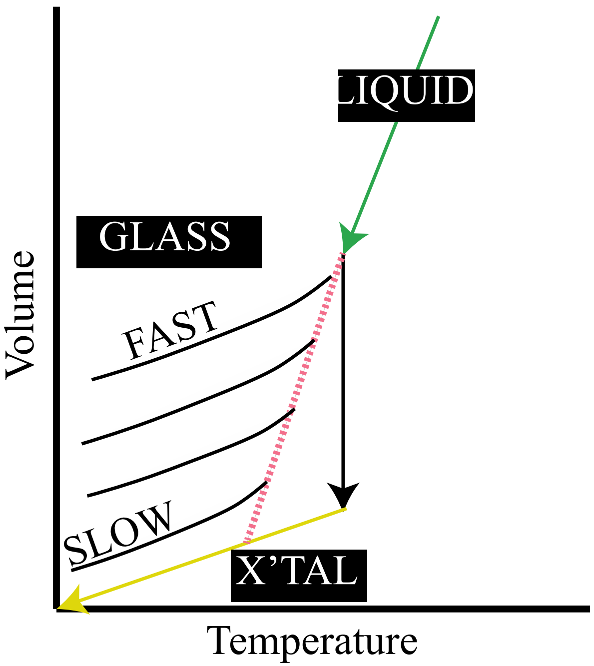
[21] Novelia and Reilly, "On Geodesics of the Rotation Group SO(3)," Reg. Chaotic Dynam. (2015).
[22] Mermin, "The homotopy groups of condensed matter," American Inst. Physics. (1978).

Hints at Restricted Dimensions:

Undercooling

All solidifying systems undercool below the melting temperature (T_M).



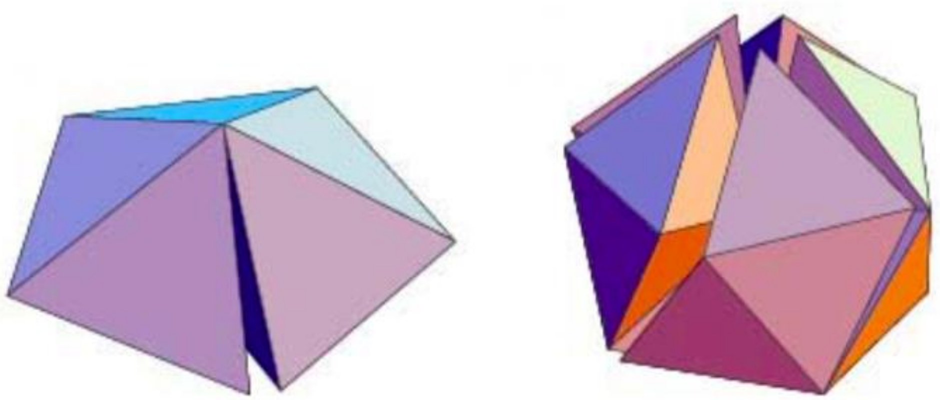


[23] Kauzmann, "The Nature of the Glassy State and the Behavior of Liquids at Low Temperatures," Nature (1948).

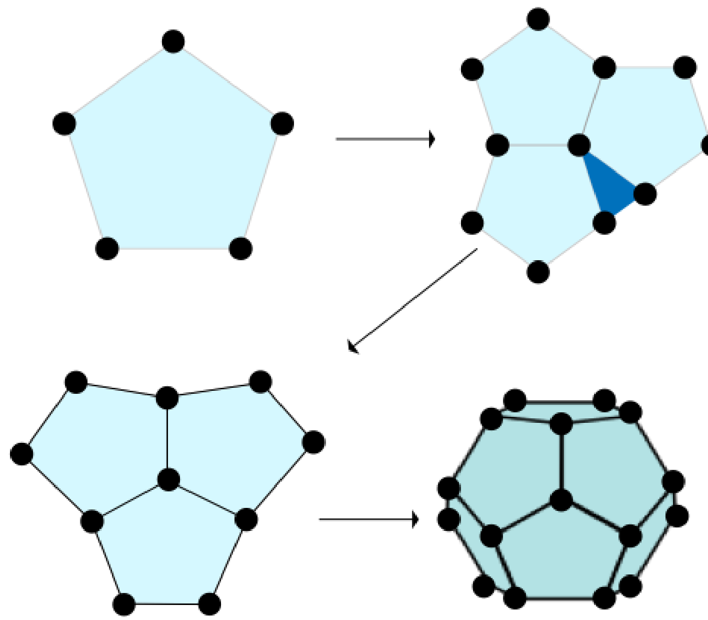
Hints at Restricted Dimensions:

Geometrical Frustration of Atomic Clusters

Undercooling related to energetic-preference for *icosahedral clustering*



icosahedral order cannot propagate, i.e., it is geometrically frustrated, gaps will remain in the structure.



Relieving geometrical frustration:

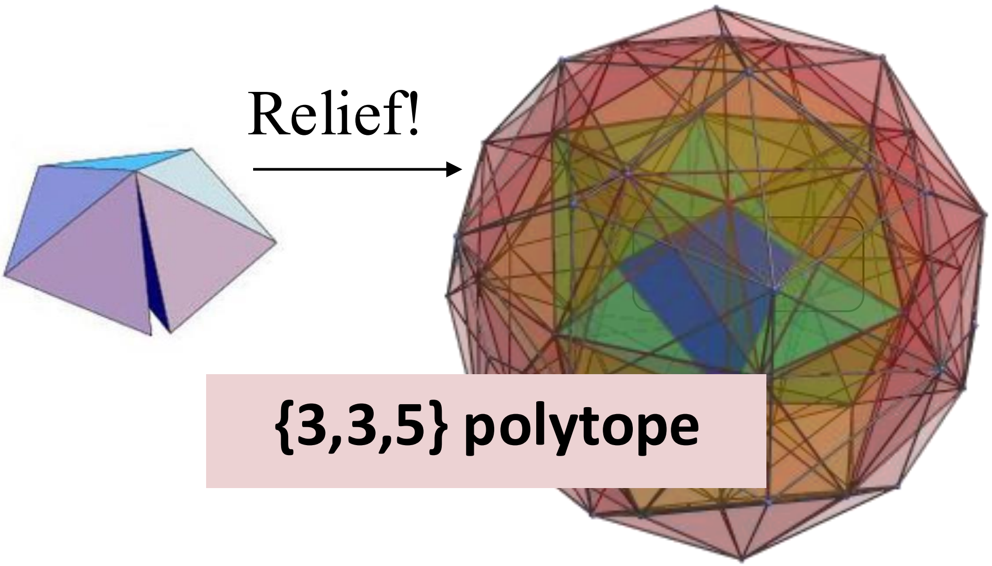
[24] F. C. Frank, "Supercooling of Liquids," Proc. Roy. Soc. London (1952).
[25] Nelson and Widom, "Symmetry, Landau theory and polytope models of glass," Nuc.Phys.B (1984).
[26] Sethna et al., "Relieving Cholesteric Frustration: The Blue Phase in a Curved Space," PRL (1983).

SU(2) Orientational Order Parameter: *Plasma of Topological defects in 3D/4D*

Below T_M , icosahedral clustering breaks **G locally** to a subgroup

$H = Y$ where Y is the icosahedral group

The *lift* of H $\in SO(3)$ into SU(2) is the $Y' = \{3,3,5\}$ polytope.



120-vertices: Each vertex is icosahedrally-coordinated

600-cells: Each cell is tetrahedral, *5 per bond*

- The order parameter *manifold* $\mathcal{M}$ is:

$$\mathcal{M} = \frac{\mathrm{G}}{\mathrm{H}} = SU(2)/Y'$$

- Topological defects are described by **homotopy groups** of $\mathcal{M}$,
  - $\pi_1(\mathcal{M}) = \mathrm{Y}'$
  - $\pi_3(S^3) = Z$

[27] Nelson, “Liquids and Glasses in Spaces of Incommensurate Curvature,” PRL (1983).
[28] M. Widom, “Icosahedral order in glass: Electronic properties” PRB (1984).
[29] M. Widom, “Short- and Long-Range Icosahedral Order in Crystals, Glass and Quasicrystals” (1988).
[30] Mermin, “The topological theory of defects in ordered media,” Rev. Mod. Phys. (1979).

Crystallization in the Lab:
π_1 *Topological Defect Binding*

As clusters in the molten undercooled liquid take on all orientations, and rotate, there is a *gas* [(+) and (-)] of angular disclination line defects (π_1)

Upon crystallization, <u>pairs of disclinations</u> bind into <u>dislocations</u>

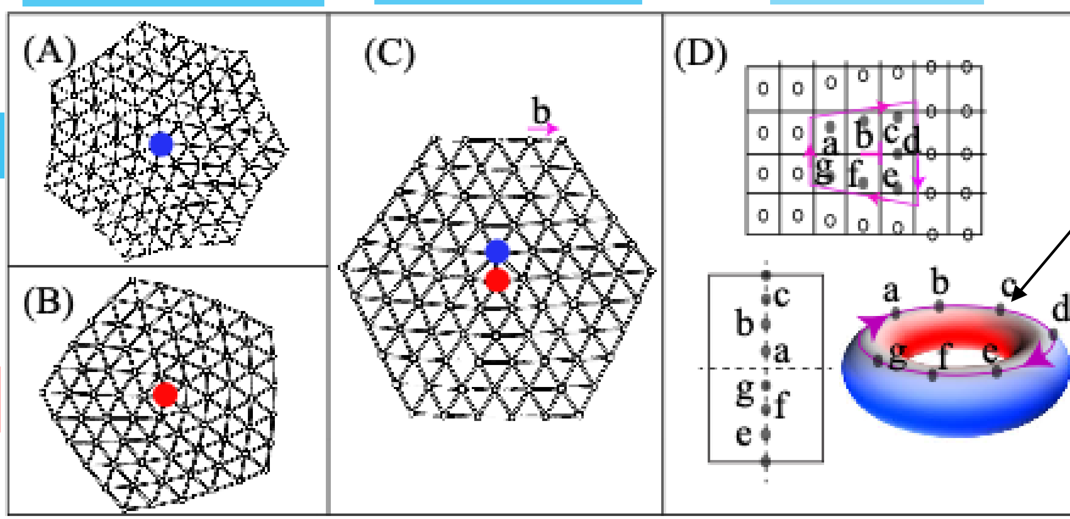
Disclinations

Dislocation

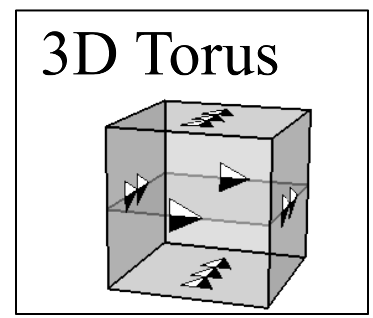
torus BZ

(-) curvature

(+) curvature



<u>zero integral curvature</u>



<u>CRYSTALLIZATION</u> is a <u>FLATTENING</u> of a system into a space-filling lattice.

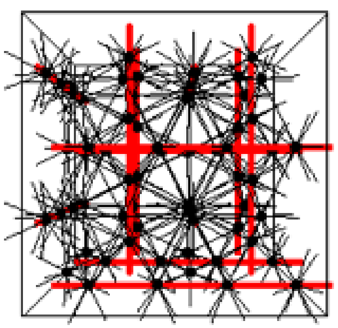
[31] Pretko, Radzihovsky. "Fracton-elasticity duality." *PRL* (2018).

Crystallization in the Lab: *Topologically close-phased (Frank-Kasper)*

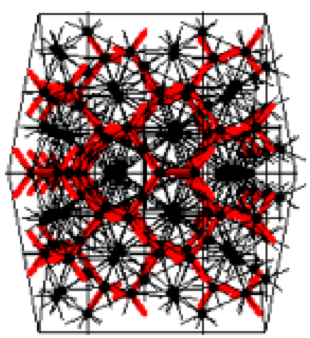
Finite geometrical frustration forces signed disclinations to persist to the ground state, in a manner that satisfies *3rd law of thermodynamics!*

Frank-Kasper TCP periodic arrangement of negative curvature disclinations $\pi_1(S^3/Y')$

A15



C15



misorientation is concentrated at signed defect cores

$\Delta\boldsymbol{\theta}_{\boldsymbol{i}} \neq \mathbf{0}$ for $\boldsymbol{i} = \mathbf{0}, \mathbf{1}, \mathbf{2}$

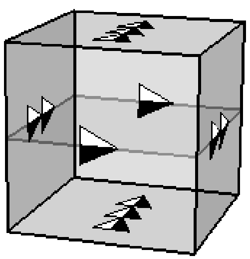
[32] Frank, Kasper. *Acta Crystallographica* 12.7 (1959)

4D/3D Crystal/Glass QPT:

Topology of "Ideal Glass Transition"

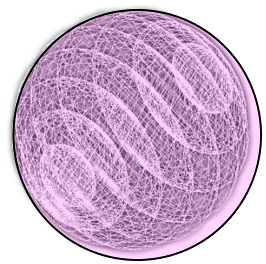
At the quantum phase transition that exists for quaternion ordered systems in restricted dimensions, there is a discontinuous change in the **genus topological invariant** of $\mathcal{M}$ which represents the ground state.

$g = 0$

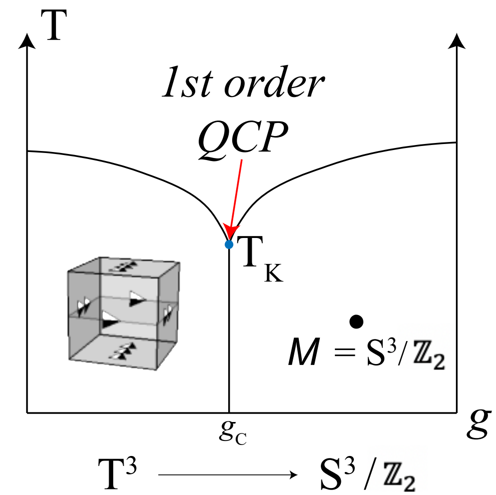


Crystal ($\boldsymbol{T}^3 \in \mathbb{R}^3$)

$g = g_C$



"Ideal Glass" ($\mathbf{S}^3 \in \mathbb{R}^3$)

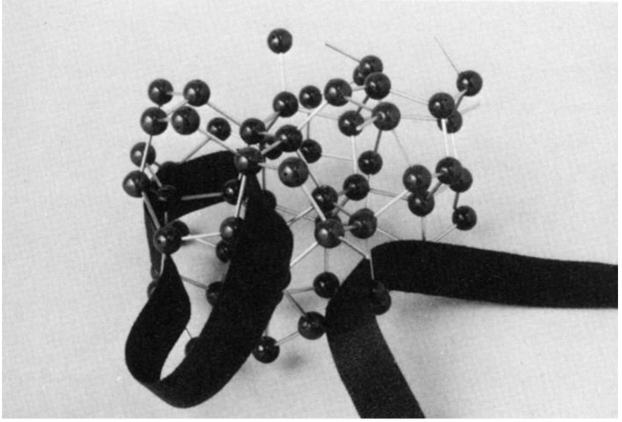


This discontinuous change in the ground state leads to a 1st order quantum critical point at the finite Kauzmann Temperature (T_K)

[33] **C. S. Gorham** and D. E. Laughlin, "Crystallization in Three-Dimensions: Defect-Driven Topological Ordering and the Role of Geometrical Frustration," Physical Review B, 99, 144106, (2019).

What about glasses?
Structure & Excitations

In topological literature, structures of glasses are considered as "*en*tangled networks of disclinations."

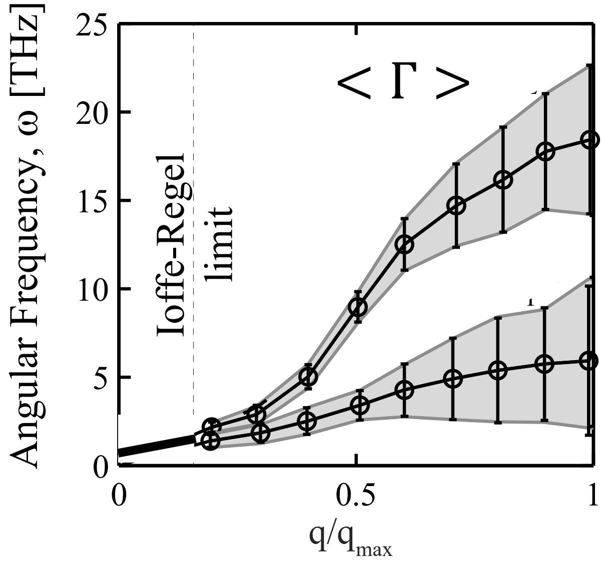


- Misorientational defects [*non-abelian disclinations and π_3 defects*] themselves form a condensate – pinning *atomic clusters*

$\mathbf{\Delta n \rightarrow 0}$

- Pinned clusters form bound states – particle-hole pairs – described by two-level tunneling systems (TLS).

$\boldsymbol{\Delta\theta_i \neq 0}$



No massless Nambu-Goldstone continuum modes.

Only gapped modes.

[34] Rivier, "Gauge theory of glass," Springer, Berlin, Heidelberg (1983).
[35] Nelson, PRL 50.13 (1983).
[36] Anderson, "Anomalous low-temperature thermal properties of glasses and spin glasses" J Theoretical Experimental Applied Physics (1971)
[37] Larkin McGaughey, "Thermal conductivity accumulation in amorphous silica and amorphous silicon," PRB (2014).

Conclusion & Future Work

In regard to – Solid states and their transport properties. Many researchers study the glass problem from 1st-principles approaches, without a foundational interpretation of the ground state, order parameter, expected excitations and implied intrinsic duality. This leaves insufficient context to interpret results.

In regard to – Condensed matter physics in general. These approaches provide the foundation for higher-dimensional generalizations:

- Quantum Hall Effects and Chern classes (conductance and flux quantum);
- Meissner & Josephson Effect equations;
- and – offer a new approach to the study of cosmology in the laboratory

OUR ARTICLES

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